Optical counterpart of the Foucault pendulum.

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The rotation of the Earth was one of the most controversial issues of natural philosophy during centuries in transition from medieval to Renaissances and afterwards. The invention of Foucault pendulum [1] did not stopped this controversies but further stimulated the new searches of the Earth motion with respect to the nonexistent "eather wind". The special relativity arose from Michelson's interferometric studies of small displacements and stars dimensions [2]. Next achievement was Saqnac discovery of the phase lag of counter propagating waves caused by rotation of the reference frame [3]. Nowadays the Sagnac effect is in the heart of the widespread rotation sensors, technically implemented as a passive fiber gyroscopes and the active laser gyros [4]. During the recent decades the activity of researchers is focused upon the photon's rotation, observable as optical vortices [5, 6]. Because of the dual photon's nature it's rotation is a very different from a classical mechanical top, used since 1817 when mechanical gyro was first realized by Johann Bohnenberger. In contrast to classical top the photon has a very remarkable difference: the projection of the angular momentum of photon on a given axis Z may have only discrete values, proportional to the Plank's constant $\hbar$. The goal of the present Letter is to show the fair experimental possibility of the Earth rotation detection with a very simple optical interferometer of less than 1 meter length based upon fundamental property of the optical photon - the discreteness of angular momentum in quantum optics [7]. In contrast to the traditional interferometers the key feature of this interferometer is the usage of the phase-conjugating mirror [8], which alters the photon's angular momentum to the exactly opposite direction [9].

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We start from the classical experiment on the detection of the optical angular momentum performed by Richard Beth in 1936 [10]. Beth transmitted a circularly polarized light through the so-called "half-wavelength" plate ($\lambda/2$ plate), suspended on quartz wire. The circularly polarized light carries the angular momentum $\pm\hbar$ per photon. Such transparent plates are made from *anisotropic* material (quartz) which changes the angular momentum of each photon to the opposite one $\mp\hbar$ during passage through the plate. In accordance with the second Newton's law and the angular momentum $\vec{L}$ conservation the plate experienced the torque $\vec{T} = \Delta\vec{L}/\Delta t$, where $(\Delta\vec{L})$ is angular momentum change during time interval Δt. The electrodynamical explanation of the torque origin is the *noncollinearity* of the electric field vector of light $\vec{E}$ and macroscopic polarization $\vec{P}dV$ (dipole moment of the volume dV) of the plate. This noncollinearity is a manifestation of anizotropy of the $\lambda/2$ plate: $\vec{T} = \epsilon_0 \int(\vec{P}\times\vec{E})dV \cong 2\cdot I\cdot \pi D_0^2/\omega$, where the 3D integral is calculated over the plate volume, I is light intensity, D_0 is the radius of the plate and $\omega_{f,b}$ is light carrier frequency. Thereby the suspending wire was twisted and a certain deflection of plate from equilibrium position was detected. In order to enhance the torque twice Beth reflected light by a traditional metallic mirror. Our proposal is to replace the traditional mirror by wavefront reversing mirror [11], which alters *orbital angular momentum* of photons [9], replace $\lambda/2$ plate to a sequence of the N image altering elements (alike Dove prism) [6] and to use higher-order optical vortices with angular momentum $\pm\ell\hbar$ per photon [7], instead of circularily polarized light. The usage of the photorefractive crystal based phase-conjugating mirror [12, 13] or else an equivalent static 3D holograms [14] as a phase-conjugating mirror (PCM) looks as the most appropriate for rotation detection purposes. Thus Beth's torsion pendulum setup is transformed into *interferometric* setup realized just recently by C.Denz with collaborators [13] (fig.1). Instead of the altering the *spin* component of photon's angular momentum, the alternation of the *orbital angular momentum* is to be performed [15]. The interference pattern which arise due to the reversed orbital angular momentum of backward reflected phase conjugated wave $E_b(t,z,r,\theta) = E^*_f(t,z,r,\theta)$ has a nontrivial geometry. This pattern is composed of the 2ℓ mutually embedded helices (fig.1):

$$|\vec{E}|^2 = |E_f + E_b|^2 \cong I(z,r,\theta,t) \sim [1 + cos[\,(\omega_f-\omega_b)t - (k_f+k_b)z + 2\ell\,\theta]] \times (r/D_0)^{2|\ell|}\, exp\,[\,-\frac{2r^2}{{D_0}^2(1+z^2/({k_{(f,b)}}^2{D_0}^4))}]\ . \qquad (1)$$

where the cylindrical coordinates (z,r,θ) are used, $k_{f,b}, \omega_{f,b}$ are the wavenumbers and frequencies of the E_f and E_b respectively, $I(z,r,\theta,t)$ is optical intensity distribution of 2ℓ intertwined helices. Apart from spirality formula (1) describes *synchronous rotation* of all 2ℓ helices around propagation axis Z with angular velocity $\omega_f - \omega_b$ [9]. The rotation will appear when frequencies of the forward E_f and backward waves E_b are different. The

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frequency splitting may be caused by angular momentum transfer from photon to *rotating* interferometer or vice versa, i.e. photon may acquire energy from rotating interferometer components or deliver energy to interferometer. This mutual exchange of energy and angular momentum between photon and Mach-Zehnder interferometer was already reported by Dholakia in 2002 [16] and the interference patterns revolving with Hz-order frequencies were recorded. In essence there is no difference, whether the single element rotates (Dove-prism [15] or $\lambda/2$ plate [16]) or entire interferometric setup is rotated as a whole. In both cases the rotational Doppler shift $\delta\omega$ will occur due to the angular momentum exchange between photon and setup. The phase-conjugating mirror will substantially simplify the implementation of such $sub-Hz$ rotation sensor because of the self-adjustment property of the PCM [8]. The perfect match of amplitudes and phases of forward and backward waves achieved in the Denz's photorefractive setup [13] have resulted in a remarkable $two-spot$ output pattern, obtained by virtue of beamsplitter BS placed at the entrance to interferometer (fig.1). Two-spot output of this vortex phase-conjugating interferometer [13] is the result of usage of the single-charged optical vortex (LG_{01}) laser beam. The two-spot output is given by the intersection of two helices belonging to interference pattern by plane surface of the beamsplitter BS. For the ℓ charged vortices [17, 18] the 2ℓ spot output pattern will rotate around propagation axis Z with angular velocity $\Omega_{rot} = \dot{\theta} = (\omega_f - \omega_b)/2\ell$. The $\delta\omega$ shift may appear due to the rotational Doppler effect entirely, provided internal PCM mechanism is static and moving internal waves are absent [9, 19]. The elementary approach based upon conservation of energy and angular momentum demonstrated by Dholakia [16] gives also the exact formula for the rotational Doppler shift (RDS) caused by rotation of PCM around propagation axis Z:

$$\delta\omega = \omega_b - \omega_f = \pm 2\ell \cdot \Omega + \frac{2\ell \cdot \hbar}{(I_{zz})_{PCM}}, \qquad (2)$$

where I_{zz} is the moment of inertia of PCM around z-axis. The second term in the right-hand size of (2) is negligible for typical masses ($m \sim g$) and sizes ($r \sim cm$) of a prisms and mirrors $\hbar/I_{zz} \sim \hbar/(m \cdot r^{-2}) \cong 10^{-27} Hz$. The frequency shift is proportional to the topological charge of the photon ℓ and this is connected with increased angular momentum change in reflection from rotating PCM ($2\ell\hbar$) and double passage through rotating Dove prism ($4\ell\hbar$). Using this physically transparent arguments we obtain expression for the *net* frequency shift for the photon, which passed twice, in forward and backward directions, through N image inverting elements [6] after reflection from the phase-conjugating mirror $\delta\omega_{net} = 4\Omega \cdot \ell(N + 1/2)$. The effect is *additive*, not *multiplicative*. The necessary condition to RDS accumulation is counter rotation of adjacent image inverting elements. The frequency shift $\delta\omega(z)$ is stepwise function of Z (fig.1): the smallest speed of helix rotation $\dot{\theta} = \Omega$ is in between PCM and first Dove prism, the largest one $\delta\omega_{net}$ is in between last Dove prism and beamsplitter BS. Consequently the modified phase conjugating interferometer [13] is capable to enhance the slow rotations of the reference frame by purely geometrical means: reflections and refractions [19].

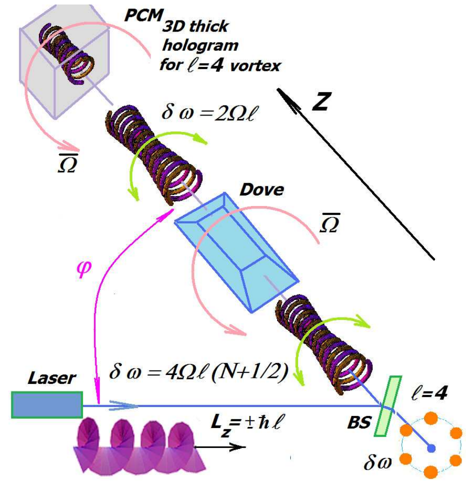

FIG. 1: (Color online) Phase-conjugating interferometer [8] aligned along the reference frame rotation axis $\vec{\Omega}$. Laser diode emits optical vortex with topological charge $\ell = 4$. The N counter rotating Dove prisms (only one shown) and PC mirror alter the photon's angular momentum and thereby the frequency shift $\delta\omega$ appears. Helical interference pattern rotation [9] is recorded by beamsplitter BS [13]. When setup is fixed on a slowly rotating platform having period of rotation $\tau \cong 10 - 100sec$ the period of rotation of interference pattern is $(N + 1/2)$ times smaller. Angle ϕ is a tilt to the horizon when the Earth rotation is measured ($\tau \cong 86400sec$).

Two following examples demonstrate 24-fold and 1440-fold magnification of the $\Omega_\oplus$ (Earth rotation). For $\ell = 4$ optical vortex having $L_z = \pm 4\hbar$ per photon the *single* Dove prism placed between BS and PCM makes possible to observe one rotation per hour of the $eight-spot$ interference pattern inside the phase-conjugating vortex interferometer (PCVI), fixed on the Earth ground or at rotating platform. For this purpose the axis of PCVI Z ought to be parallel (antiparallel) to the vector of angular frequency $\vec{\Omega}_\oplus$. This happens at the North and South poles for vertical PCVI or when PCVI is inclined from vertical towards North at the angle $\pi/2 - \phi$, where ϕ is geographical latitude. Obviously the effect will be zero when Z-axis is aligned along East to West latitude line. Thus tilt of the PCVI with respect to horizont should be 55.4^o in Moscow [8], 53^o in Munster [13] and 55.5^o in Glasgow and Fife [15, 16]. With the $\ell = 6$-charged optical vortex and 60 Dove prisms placed in between PCM and BS, the $twelve-spot$ interference pattern spots will cross the detector window once a minute. The above outlined interferometric setup may be realized with already tested routine optical components: laser diodes with Ghz linewidth (or about one meter coherence length), photorefractive $BaTiO_3$ [13] and SBN phase conjugators,

thick holograms [14] or even optical *loops* [19].

The nontrivial feature of PC mirror is a so-called *time reversal* property [11], which means that the optical wave $E_{f,b}(\vec{r},t)$, reflected from PCM propagates in such a way, that it passes all configurations of incident wave $E_f(\vec{r},t)$ in reverse sequence. Thus both waves, E_f and $E_b = E_f^*$ have the identical in space distributions of intensity and coincided wavefronts (conjugated wavefronts). The phase-congugating mirror will feel the torque because of severe internal anisotropy [9]. The anisotropy appears due to the helical interference pattern (1) which exists both *inside* and outside of PC mirror. Inside mirror the 2ℓ helix exists in the forms of excessive charge wave (photorefractive crystals [12, 13]), static 3D hologram inside thick photographic plate ([14]), spatially modulated index due to inhomogeneous orientation of chiral molecules in a liquid-crystals ([11]). This internal helicity [9] of PCM is the cause of optical torque $\vec{T}$ and consequently of the rotational Doppler shift $\delta\omega$.

It is instructive to stress the point that the optical torque $\vec{T}$ plays the crucial role in proposed PCM interferometric experiment but in an apparently different context than in the Beth's classical experiment [10]. The $\lambda/2$ plate deflection was *static* in his experiment, exactly in the spirit of Coulomb [20] and Cavendish [21] experiments with torsion pendulums. The rotation, induced by optical torque, was intentionally stopped with the aid of the quartz suspension wire. Thus a very important consequence of angular momentum transfer, namely rotational Doppler shift was completely excluded from consideration. Noteworthy, Beth discussed his experiment with A.Einstein, B.Podolsky and A.Kastler [10]. Had they have under the hand the narrow linewidth laser sources, they could discuss the back action of the rotating $\lambda/2$ plate upon photon and to detect, in principle, a Hz- order optical frequency shift, as it was done in 1990-th [15]. Noteworthy even though the PCVI hardware existed in 1980 yet [8], the hidden helicity of the phase conjugating mirrors was not studied properly until now [9, 13, 19] even though the sole exception do exists [22].

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